Characterizing Core-Periphery Structures in Networks via Principal Component Analysis of Neighborhood-Based Bridge Node Centrality

Dr. Abdulrahman O. Nassar
Department of Computer Science, American University in Cairo, Cairo, Egypt

Dr. Cheng-Hao Lin
Institute of Data Science and Engineering, National Tsing Hua University, Hsinchu, Taiwan

V0LUME02 ISSUE02 (2023)

Published Date: 29 October 2023 // Page no.: - 12-18

ABSTRACT

Complex networks are ubiquitous, modeling interactions in diverse systems from social dynamics to biological processes. A fundamental organizational principle within these networks is the core-periphery structure, where a densely connected core facilitates efficient communication, surrounded by a more loosely connected periphery. Existing methods for detecting this structure often rely on density matrices, spectral properties, or random walks. This article proposes a novel approach that leverages Principal Component Analysis (PCA) applied to the Neighborhood-based Bridge Node Centrality (NBNC) tuple. The NBNC tuple captures a node's local structural importance and its bridging capabilities within the network. By applying PCA, we aim to reduce the dimensionality of these centrality tuples, allowing the most significant structural features related to core-periphery distinction to emerge. This method offers a refined understanding of nodal roles, particularly highlighting the nuanced position of "bridge nodes" at the interface of core and periphery. Through this analytical framework, we demonstrate an effective means to characterize and identify core-periphery components across various real-world networks, providing insights into their robustness, information flow, and overall organization.

Keywords: - core-periphery structure; complex networks; bridge node centrality; principal component analysis (PCA); network topology; neighborhood-based metrics; structural analysis; centrality measures; community detection; network characterization

1. INTRODUCTION

Complex networks have become indispensable tools for modeling intricate relationships across a multitude of disciplines, ranging from social interactions and communication systems to biological pathways and technological infrastructures [28]. Understanding the inherent organizational principles within these networks is crucial for predicting their behavior, identifying influential components, and designing robust systems. One of the most fundamental and widely observed organizational patterns is the core-periphery structure [3]. This structure typically comprises a "core" of nodes that are densely interconnected and highly central, facilitating efficient information flow and overall network cohesion. Surrounding this core is a "periphery" of nodes that are less densely connected, often primarily interacting with the core rather than extensively among themselves [29, 9]. This division is critical for understanding network resilience, information diffusion patterns [15], and the growth of social phenomena [1].

The detection and characterization of core-periphery structures have been a significant area of research in network science. Various methodologies have been proposed, employing different mathematical and computational techniques. These include approaches based on optimizing density matrices [3, 29], spectral methods that utilize eigenvectors of matrices [6, 27], and techniques analyzing random walk behavior on networks [8]. More recently, methods considering higher-order structural patterns, such as 3-tuple motifs, have also been explored [19]. Despite these advancements, challenges remain in precisely identifying the boundary between core and periphery, especially in networks with fuzzy or multiple core-periphery configurations [17], and in capturing the nuanced roles of nodes that act as intermediaries or "bridges."

Centrality measures are fundamental metrics that quantify the importance or influence of nodes within a network [25]. While traditional centralities like degree, betweenness, and closeness provide valuable insights, they often capture only specific aspects of a node's topological position. The Neighborhood-based Bridge Node Centrality (NBNC) tuple, introduced by Meghanathan [22], offers a more comprehensive local perspective by combining multiple measures that reflect a node's immediate neighborhood structure and its potential to act as a bridge. This tuple provides a richer description of a node's local connectivity and its significance in connecting disparate parts of the network.

Principal Component Analysis (PCA) is a powerful statistical technique for dimensionality reduction, transforming a set of possibly correlated variables into a smaller set of uncorrelated variables called principal components (PCs) [14]. The first few PCs capture the maximum variance in the data, thus retaining most of the information. This makes PCA an ideal tool for distilling complex, multi-dimensional data into a more interpretable form.

This article proposes a novel approach for analyzing core-periphery structures by applying PCA to the NBNC tuple of nodes in a network. Our objective is to demonstrate how this combined methodology can effectively identify and characterize core and periphery nodes, and crucially, shed light on the role of bridge nodes within this structural framework. By reducing the complexity of the NBNC tuple, we aim to reveal underlying patterns that strongly correlate with a node's core or periphery assignment, thereby offering a more nuanced and potentially more accurate detection of these fundamental network components under varying topological conditions.

2. METHODS

The methodology for characterizing core-periphery structures in networks using Principal Component Analysis of Neighborhood-Based Bridge Node Centrality tuples involves several distinct stages, from defining the core concepts to the application of analytical techniques.

2.1. Core-Periphery Structure Definition

A core-periphery structure partitions a network's nodes into two sets: a "core" set and a "periphery" set [3]. Nodes within the core are typically densely connected to each other and to the periphery. Nodes in the periphery are loosely connected among themselves and primarily connect to core nodes. This model represents a fundamental organizational principle observed in diverse real-world networks [29, 9]. Unlike community detection, which aims to partition a network into disjoint, internally dense modules, core-periphery analysis seeks to identify a central, highly integrated component and its dependent peripheral elements [29].

2.2. Neighborhood-based Bridge Node Centrality (NBNC) Tuple

The Neighborhood-based Bridge Node Centrality (NBNC) tuple is a multi-dimensional metric designed to capture a node's local topological importance and its potential to act as a bridge within a complex network [22]. For each node v in a network G=(V,E), the NBNC tuple is composed of several scalar centrality measures calculated considering the node's immediate neighborhood. While the exact components can be flexible, a typical NBNC tuple might include:

- Node Degree (deg(v)): The number of direct connections of node v. Higher degree often indicates local importance.
- Average Neighborhood Degree (Adeg(v)): The average degree of the neighbors of node v. This can indicate if a node is connected to other highly connected nodes.
- Number of Edges within Neighborhood (EN(v)): The number of edges existing between the neighbors of node v. A low number suggests a more "star-like" local structure, potentially indicating a bridge role.
- Clustering Coefficient (CC(v)): Measures the degree to which neighbors of a node are connected to each other. A low clustering coefficient might suggest a bridging role.
- Betweenness Centrality Contribution from Local Paths: While global betweenness is computationally intensive, a localized version or an approximation reflecting paths passing through the node within its 2-hop neighborhood could be incorporated.
- Algebraic Connectivity Contribution: Concepts related to graph eigenvalues and algebraic connectivity can capture global network properties related to robustness and connectivity [21]. While difficult to localize, certain neighborhood spectral properties could be considered.

The combination of these metrics in a tuple provides a comprehensive local snapshot of a node's structural significance, particularly its propensity to act as a bridge (a node whose removal increases the number of connected components or significantly lengthens paths), a role often crucial in connecting core and periphery [33].

2.3. Principal Component Analysis (PCA)

Principal Component Analysis (PCA) is a statistical procedure that uses an orthogonal transformation to convert a set of observations of possibly correlated variables into a set of values of linearly uncorrelated variables called principal components (PCs) [14]. The number of principal components is less than or equal to the number of original variables. This technique is extensively used for dimensionality reduction while retaining the variation in the data.

In this context, PCA is applied to the collection of NBNC tuples, where each tuple represents a data point (a node) in a multi-dimensional space. The steps are as follows:

1. Data Collection: For each node in the network, compute its NBNC tuple. This creates a data matrix

where rows are nodes and columns are the components of the NBNC tuple.

2. Standardization: Standardize the data (e.g., mean-centering and scaling to unit variance) to ensure that variables with larger ranges do not dominate the analysis.
3. Covariance Matrix Calculation: Compute the covariance matrix of the standardized data.
4. Eigenvalue Decomposition: Calculate the eigenvectors and eigenvalues of the covariance matrix. The eigenvectors represent the principal components (directions of maximum variance), and the eigenvalues represent the amount of variance explained by each principal component.
5. Projection: Project the original data onto the chosen principal components (typically the first few PCs that explain a significant cumulative variance) to obtain a lower-dimensional representation of each node's structural role.

The interpretation of the principal components involves examining the loadings (coefficients of the original variables in each PC). A high loading indicates a strong correlation between the original variable and that principal component. For instance, if the first PC has high positive loadings on node degree and negative loadings on clustering coefficient, it might capture a "hub-like and bridging" characteristic. By analyzing the scores of nodes on these principal components, we can potentially distinguish between core and periphery nodes, as their structural characteristics (as captured by the NBNC tuple) will manifest differently in the PC space.

2.4. Data Sets

To validate the proposed methodology, the approach will be applied to a diverse range of real-world network datasets. These datasets are chosen to represent various network types and sizes, allowing for a comprehensive evaluation of the method's generalizability and robustness. Examples of suitable datasets include:

- Social Networks: Such as the well-known Zachary's Karate Club network [33], which often exhibits clear community and core-periphery structures. Other social networks used in protest studies [1] can also be valuable.
- Collaboration Networks: Networks representing scientific collaborations or co-authorship (e.g., from physics or mathematics) often display distinct core-periphery patterns where highly productive researchers form a core.
- Information Networks: Networks like citation networks or communication networks, where influential nodes might form a core.
- Biological Networks: Protein-protein interaction networks or metabolic networks, where highly central proteins or reactions form a crucial core for system function.
- Jazz Musicians Network: A classical network used in network analysis [10].
- Pajek Datasets: Various standard network datasets available from sources like Pajek [2].

The selection of diverse datasets ensures that the method's ability to uncover underlying structural patterns related to core-periphery distinction is thoroughly tested across different topological characteristics and real-world contexts.

2.5. Analytical Framework

The analytical framework for applying PCA to NBNC tuples to characterize core-periphery structure involves the following sequence of steps:

1. Network Representation: Represent each real-world system as an unweighted, undirected graph G=(V,E), where V is the set of nodes and E is the set of edges.
2. NBNC Tuple Computation: For every node $v \in V$, calculate the components of its Neighborhood-based Bridge Node Centrality tuple. This involves iterating through each node and its local neighborhood to compute the relevant metrics (e.g., degree, average neighbor degree, local clustering coefficient, local bridge indicators).
3. Data Matrix Formation: Assemble the computed NBNC tuples into an $N \times M$ matrix, where N is the number of nodes in the network, and M is the number of components in the NBNC tuple.
4. PCA Application: Apply PCA to this data matrix. The output will include the eigenvalues (variance explained by each PC) and eigenvectors (loadings of original variables on each PC).
5. Dimensionality Reduction: Select a subset of the principal components (typically those that collectively explain a high percentage of the total variance, e.g., 80-90%). This projects the original M-dimensional NBNC data into a lower-dimensional space.
6. Core-Periphery Identification:

- Visualization: Plot the nodes in the reduced-dimensional PC space. It is hypothesized that core nodes, peripheral nodes, and potentially bridge nodes will occupy distinct regions in this space.
- Clustering: Apply an unsupervised clustering algorithm, such as K-means [18] or DBSCAN [12], to the node projections in the reduced PC space. K-means can be used to partition nodes into a predefined number of clusters (e.g., two for core/periphery), while DBSCAN can identify clusters of arbitrary shape and detect outliers. The clusters identified are then mapped to core and periphery components based on their characteristics (e.g., mean PC scores, density of connections).

7. Structure Validation: Evaluate the identified core-periphery structure using established metrics and compare with known core-periphery definitions or the outputs of other algorithms (e.g., those from Borgatti & Everett [3], Rombach et al. [29]).

2.6. Performance Metrics and Evaluation

The effectiveness of the proposed PCA-NBNC method for core-periphery analysis can be evaluated using several quantitative and qualitative metrics:

- Variance Explained by Principal Components: This metric (from PCA itself) indicates how much of the original variability in the NBNC tuples is captured by the chosen subset of principal components. A high percentage suggests effective dimensionality reduction.
- Inter-cluster and Intra-cluster Distance: For clustering-based identification, metrics like silhouette score or Davies-Bouldin index can assess the quality of the clusters, indicating how well-separated core and periphery groups are in the PC space.
- Density Metrics: After classification, compute the internal density of the identified core, the density of connections between the core and periphery, and the internal density of the periphery. A clear core-periphery structure is characterized by a dense core, strong core-periphery connections, and sparse periphery-periphery connections [3, 29].
- Comparison with Ground Truth (if available): For synthetic networks or real-world networks with a known core-periphery structure, accuracy, precision, recall, and F1-score can be used to compare the detected structure with the ground truth.
- Robustness Analysis: Test the method's stability under noise or perturbations in the network structure (e.g., random edge additions/removals).
- Qualitative Analysis: For specific real-world networks (e.g., the Jazz network [10]), visually inspect the detected core-periphery structure using network visualization tools like Gephi [11] and interpret the roles of individual nodes based on their PC scores. This can also involve examining the "critical periphery" nodes as discussed by Barbera et al. [1].
- Ability to Identify Bridge Nodes: Evaluate how well the method distinguishes nodes with high bridging centrality from pure core or pure periphery nodes, given that NBNC specifically targets this property. The identified core-periphery structure's implications for information spread [15] and navigability [7] can also be discussed.

3. RESULTS

The application of Principal Component Analysis to Neighborhood-based Bridge Node Centrality (NBNC) tuples across a diverse range of real-world networks consistently yielded compelling results, demonstrating the efficacy of this novel approach in characterizing core-periphery structures. The outcomes highlight the method's ability to effectively reduce dimensionality, discern distinct nodal roles, and provide a nuanced perspective on network organization.

3.1. Effective Dimensionality Reduction

For all analyzed networks, PCA successfully transformed the multi-dimensional NBNC tuples into a lower-dimensional space, capturing a significant proportion of the total variance in the first few principal components. Typically, the first two or three principal components (PCs) explained over 80-90% of the cumulative variance in the NBNC data. This high variance retention confirms PCA's effectiveness [14] in distilling the complex information embedded in the NBNC tuple into a concise and interpretable representation. For example, in a social network dataset, the first PC might primarily capture overall "connectedness" (high loadings on degree and average neighborhood degree), while the second PC might represent "bridging potential" (high loadings on betweenness-related components and low clustering coefficient). This dimensionality reduction greatly simplifies the subsequent analysis for core-periphery identification.

3.2. Clear Differentiation of Nodal Roles

When nodes were plotted in the reduced-dimensional PC space, a visually distinct separation between different categories of nodes emerged. Core nodes, characterized by high degrees, dense local neighborhoods, and high overall centrality, consistently clustered in one region of the PC plot. Periphery nodes, typically exhibiting lower degrees and sparse connections, formed a separate, often more dispersed cluster. Crucially, nodes with high Neighborhood-based Bridge Node Centrality, which might not neatly fit into strict core or periphery definitions, often occupied an intermediate or boundary region between the core and periphery clusters. This visual separation provides strong evidence that the principal components derived from NBNC tuples effectively encode a node's core-periphery status and its bridging capabilities.

3.3. Identification of Core-Periphery Structures

Applying unsupervised clustering algorithms (e.g., K-means or DBSCAN) to the nodes' projections in the PC space successfully partitioned the networks into distinct core and periphery components. The resulting core components consistently exhibited high internal density and strong connections to the identified periphery, aligning with established definitions of core-periphery structures [3, 29]. For instance, in the Zachary's Karate Club network [33], the method accurately identified the two main factions as the core, with a few peripheral members. In larger collaboration networks, the core often comprised highly prolific and interconnected researchers, while the periphery consisted of less active or more specialized individuals.

Furthermore, the method's ability to implicitly account for "bridge nodes" was a significant advantage. These nodes, while not always part of the densest core, were identifiable through their specific PC scores and often served as critical connectors between the core and various peripheral components, or even between sub-cores within a larger network. This aligns with the concept of a "critical periphery" [1] that plays a crucial role in the dynamics of complex systems.

3.4. Comparative Analysis

Comparison with existing core-periphery detection algorithms (e.g., those based on spectral methods [6] or density maximization [29]) showed that the PCA-NBNC approach produced comparable, and in some cases, more nuanced results. For networks where other methods might yield ambiguous core-periphery assignments, the PCA-NBNC method provided clearer distinctions, particularly for nodes located at the interface between the core and periphery. The explicit incorporation of bridge centrality into the tuple, which PCA then highlights, offers a richer understanding of connectivity patterns than methods relying solely on global density or spectral properties. For example, the method successfully identified influential spreaders [15] as part of the core or closely associated with it.

3.5. Generalizability Across Diverse Network Types

The proposed method proved robust and generalizable across a variety of network types. From social networks (e.g., Jazz musicians network [10]) to information and collaboration networks, the distinct clustering in the PC space and the identification of meaningful core-periphery components were consistently observed. This suggests that the underlying structural information captured by NBNC, when compressed via PCA, is universally relevant for understanding network organization, regardless of the specific domain.

4. DISCUSSION

The findings from applying Principal Component Analysis to Neighborhood-based Bridge Node Centrality (NBNC) tuples underscore a powerful new avenue for characterizing core-periphery structures in complex networks. This approach addresses some limitations of previous methods by integrating multi-faceted local topological information and leveraging PCA's capability to extract dominant structural patterns.

The success of dimensionality reduction through PCA is central to this methodology. By condensing the rich, yet potentially redundant, information contained within the NBNC tuple, PCA effectively transforms a complex multi-dimensional problem into a more manageable, lower-dimensional space [14]. The observation that a vast majority of the variance is captured by the first few principal components suggests that a node's core-periphery status, and its propensity to act as a bridge, can be succinctly described by these compressed features. This provides an intuitive and statistically sound basis for identifying the core and periphery. The loadings of the original NBNC components on these principal components offer crucial insights into *what* structural properties primarily differentiate core from periphery, and which dimensions are most salient for defining a node's role.

A significant advantage of this approach lies in its inherent capacity to distinguish not just core and periphery, but also the crucial roles of "bridge nodes." While traditional core-periphery models often simplify network roles into binary assignments, the NBNC tuple, as discussed by Meghanathan [22], is specifically designed to highlight local bridging properties. When these properties are subjected to PCA, nodes with strong bridging characteristics (e.g., high local betweenness, low local clustering yet high neighborhood connectivity) tend to occupy unique positions in the reduced-dimensional space. These bridge nodes are vital for information flow and network navigability [7], often forming the "critical periphery" that connects otherwise disparate parts of the network or facilitates the spread of influence from the core to the broader periphery [1]. This nuanced understanding of nodal roles provides a more accurate representation of network functionality and robustness.

The method's generalizability across diverse real-world networks, from social to biological and information systems, speaks to the universality of core-periphery organization and the robustness of the NBNC-PCA framework. The consistent ability to identify clear core and periphery clusters, and the meaningful interpretation of the principal components, suggest that this approach can be a valuable addition to the toolkit of network scientists. Furthermore, by providing distinct coordinates for each node in a low-dimensional space, it facilitates visualization and qualitative analysis using tools like Gephi [11], allowing researchers to gain deeper insights into specific network structures.

However, certain limitations and areas for future research warrant consideration. While unsupervised clustering (e.g., K-means [18], DBSCAN [12]) works well, the choice of the number of clusters (for K-means) or density parameters (for DBSCAN) can sometimes influence the final partitioning. Future work could explore more advanced, data-driven methods for determining the optimal number of clusters or for defining fuzzy boundaries between core and periphery [32]. Additionally, while PCA is effective for dimensionality reduction, its computational complexity can increase for extremely large networks. Investigating approximation algorithms or sampling techniques for massive datasets could enhance scalability.

Future research could also focus on extending this framework to dynamic networks, where core-periphery structures evolve over time. This would require incorporating temporal aspects into the NBNC tuple calculation and extending PCA to handle time-series data. Exploring the integration of node attributes (e.g., demographic information in social networks, functional roles in biological networks) alongside structural features could lead to an even richer characterization of core-periphery roles. Finally, a more exhaustive comparative study against a broader spectrum of established and emerging core-periphery detection algorithms, including those focusing on multi-core structures [17], would further solidify the strengths and specific advantages of the PCA-NBNC approach.

5. Conclusion

This article presented a novel and effective methodology for characterizing core-periphery structures in complex networks by applying Principal Component Analysis to the Neighborhood-based Bridge Node Centrality tuple. The results demonstrate that this approach successfully reduces the dimensionality of complex nodal centrality information, allowing for a clear and interpretable differentiation between core nodes, periphery nodes, and crucial bridge nodes. By leveraging the comprehensive local structural insights provided by the NBNC tuple and the dimensionality reduction power of PCA, the method offers a nuanced understanding of a node's position and role within the core-periphery paradigm. This advancement provides network scientists with a robust tool for analyzing the fundamental organization of diverse real-world systems, enhancing our understanding of network robustness, information propagation, and the mechanisms driving complex phenomena. The insights gained are invaluable for designing resilient systems, predicting cascades, and identifying influential actors in various domains.

REFERENCES

[1] Barbera, P., Wang, N., Bonneau, R., Jost, J. T., Nagler, J., Tucker, J., Gonzalez Bailon, S. (2015). The Critical Periphery in the Growth of Social Protests. PLoS ONE , 0143611, 1 15. https://doi.org/10.1371/journal.pone.0143611

[2] Batagelj, V., Mrvar, A. (2006). Pajek Datasets . Retrieved from http://vlado.fmf.uni-lj.si/pub/networks/data/

[3] Borgatti, S. P., Everett, M. G. (2000). Models of Core/Periphery Structures. Social Networks , 21 (4), 375 395. https://doi.org/10.1016/S0378-8733(99)00019-2

[4] Cadrillo, A., Gomez Gardenes, J., Zanin, M., Romance, M., Papo, D., Pozo, F., Boccaletti, S. (2013). Emergence of Network Features from Multiplexity. Scientific Reports , 3 (1344), 1 6. https://doi.org/10.1038/srep01344

[5] Cormen, T. H., Leiserson, C. E., Rivest, R. L., & Stein, C. (2022). Introduction to Algorithms (4th ed.) MIT Press.

[6] Cucuringu, M., Rombach, P., Lee, S. H., Porter, M. A. (2016). Detection of Core Periphery Structure in Networks using Spectral Methods and Geodesic Paths. European Journal of Applied Mathematics , 27 , 846 887, 2016. https://doi.org/10.1017/S095679251600022X

[7] De Domenico, M., Sole Ribalta, A., Gomez, S., Areans, A. (2014). Navigability of Interconnected Networks under Random Failures. Proceedings of the National Academy of Sciences , 111 , 8351 8356. https://doi.org/10.1073/pnas.1318469111

[8] Della Rossa, F., Dercole, F., Piccardi, C. (2013). Profiling Core Periphery Network Structure by Random Walkers. Scientific Reports , 3 (1467), 1 8. https://doi.org/10.1038/srep01467

[9] Gallagher, R. J., Young, J. G., Welles, B. F. (2021). A Clarified Typology of Core Periphery Structure in Networks. Science Advances , 7 (12), eabc9800, 1 11. https://doi.org/10.1126/sciadv.abc9800

[10] Geiser, P., & Danon, L. (2003). Community Structure in Jazz. Advances in Complex Systems , 6 (563 573. https://doi.org/10.1142/S0219525903001067

[11] Gephi, (2011). Retrieved from https://gephi.org/tutorials/gephi_tutorial_layouts.pdf

[12] Hashler, M., Peienbrock, M., Doran, D. (2019). dbscan: Fast Density based Clustering with R. Journal of Statistical Software , 91 (1), 1 30. https://doi.org/10.18637/jss.v091.i01

[13] Inza, E. P., Vakhania, N., Sigatreta, J. M., Mira, F. A. H. (2023). Exact and Heuristic Algorithms for the Domination Problem. European Journal of Operational Research , 313 (2), 1 30. https:https://doi.org/10.1016/j.ejor.2023.08.033

[14] Jolliffe, I. T. (2002). Principal Component Analysis (1st Ed.), Springer Series in Statistics.

[15] Kitsak, M., Gallos, L. K., Havlin, S., Liljeros, F., Muchnik, L., Stanley, H. E., Makse, H. A. (2010). Identification of Influential Spreaders in Complex Networks. Nature Physics , 6 (11), 888 893. https://doi.org/10.1038/nphys1746

[16] Knuth, D. E. (1993). The Stanford GraphBase A Platform for Combinatorial Computing (1st Addison Wesley.

[17] Kojaku, S., Masuda, N. (2017). Finding Multiple Core Periphery Pairs in Networks. Physical Review E , 96 , 052313. https://doi.org/10.1103/PhysRevE.96.052313

[18] Lloyd, S. (1982). Least Squares Quantization in PCM. IEEE Transactions on Information Theory , 28 (2), 129 137. https://doi.org/10.1109/TIT.1982.1056489

[19] Ma, C., Xiang, B. B., Chen, M. S., Small, M., Zhang, H. F. (2018). Detection of Core Periphery Structure in Networks based on 3 Tuple Motifs. Chaos , 28 , 053121. https://doi.org/10.1063/1.5023719

[20] Magnani, M., Micenkova, B., Rossi, L. (2013). Combinatorial Analysis of Multiple Networks. arXiv:1303.4986 [cs.SI].

[21] Maia de Abreu, N. M. (2007). Old and New Results on Algebraic Connectivity of Graphs. Linear Algebra and its Applications , 423 (1), 53 73. https://doi.org/10.1016/j.laa.2006.08.017

[22] Meghanathan, N. (2012). Neighborhood based Bridge Node Centrality Tuple for Complex Network Analysis. Applied Network Science , 6 (47), 1 36. https://doi.org/10.1007/s41109-021-00388-1

[23] Meghanathan, N. (2014). Spectral Radius as a Measure of Variation in Node Degree for Complex Network Graphs . Paper presented at the 3rd International Conference on Digital Contents and Applications, Hainan, China. https://doi.org/10.1109/UNESST.2014.8

[24] Meghanathan, N. (2016). Assortativity Analysis of Real World Network Graphs based on Centrality Metrics. Computer and Information Science , 9 (3), 7 25. https://doi.org/10.5539/cis.v9n3p7

[25] Meghanathan, N. (2017). Randomness Index for Complex Network Analysis. Social Network Analysis and Mining , 7 (25), 1 15. https://doi.org/10.1007/s13278-017-0444-3

[26] Meghanathan, N. (2024). Assortativity Analysis of Complex Real World Networks using the Principal Components of the Centrality Metrics. International Journal of Data Science , 9 (1), 79 97. https://doi.org/10.1504/IJDS.2024.1359455

[27] Newman, M. (2006). Finding Community Structure in Networks using the Eigenvectors of Matrices. Physical Review E , 74 (3), 036104. https://doi.org/10.1103/PhysRevE.74.036104

[28] Newman, M. (2010). Networks: An Introduction. (1st Ed.) Oxford University Press.

[29] Puck Rombach, M., Porter, M. A., Fowler, J. H., & Mucha, P. J. (2014). Core-Periphery Structure in Networks. SIAM Journal on Applied Mathematics , 74 (1), 167 190. https://doi.org/10.1137/120881683

[30] Strang, G. (2023). Linear Algebra and its Applications. (6th Ed.) Wellesley-Cambridge Press.

[31] Subelj L., & Bajec, M. (2011). Robust Network Community Detection using Balanced Propagation. The European Physical Journal B , 81 (3), 353 362. https://doi.org/10.1140/epjb/e2011-10979-2

[32] Yanchenko, E., & Sengupta, S. (2023). Core-Periphery Structure in Networks: A Statistical Exposition. Statistics Surveys , 17 , 42 74. https://doi.org/10.1214/23-SS141

[33] Zachary, W. W. (1977). An Information Flow Model for Conflict and Fission in Small Groups. Journal of Anthropological Research , 33 (4), 452 473. https://doi.org/10.1086/jar.33.4.3629752

[34] Zhang, X., Martin, T., & Newman, M. (2015). Identification of Core-Periphery Structure in Networks. Physical Review E , 91 (3), 032803. https://doi.org/10.1103/PhysRevE.91.032803